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DYNAMIC RESPONSE ANALYSIS OF TUBE ARRAY IN PARTIALLY FILLED CALANDRIA

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SUMMARY

An efficient procedure is presented for dynamic response analysis of horizontal tube array in partially filled calandria including hydrodynamic interaction effects. The procedure is general enough to consider the transfer of energy between the fluid-coupled tubes, and effects of moderator sloshing on the magnitude and the distribution of hydrodynamic forces. It has been demonstrated that the conventional added mass approach fails to represent behaviour of the tube array correctly, and it is therefore necessary to consider the flexibility of all the tubes along two directions simultaneously. The procedure presented can simulate the added damping effects due to hydrodynamic interaction. The possible use of a tuned damper tube is suggested for controlling sloshing effects for tube array in a calandria where tube frequencies and sloshing frequencies are closely spaced. The presence of surface damping in the tuned tube further brings down the response, and the suggested procedure can be effectively used to control unwarranted sloshing effects.

KEY WORDS: tube array; dynamic response; calandria; tuned damper tube; fluid–structure interaction; hydrodynamic effects

INTRODUCTION

Dynamic response analysis of horizontal tube array in partially filled calandria must recognize the hydrodynamic interaction forces and modifications in their vibration properties to ensure non-vulnerability during design seismic event. The transfer of energy between the fluid-coupled tubes and the resulting significant effects on the amplitude of tube motion requires coupled solution of the equations of motion for all tubes. Additionally, because of hydrodynamic coupling, the resultant hydrodynamic forces on tubes are not in phase or in the direction of applied excitation,[1,2] thus requiring simultaneous solution for multi-component excitation. In partially filled calandria, sloshing of moderator further complicates the fluid-structure interaction problem. The magnitude and the distribution of hydrodynamic forces also depend on the extent

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of tuning between the frequencies of tubes and the sloshing frequencies of moderator contained in calandria.[3]

For multiple rigid submerged tubes under seismic excitations, the concept of self-added and coupled-added mass based on potential theory has been applied, although the experimental confirmation is far less extensive for large tube array.[4] An analytical procedure is required to determine the design values of earthquake responses for large tube array including the effects of moderator sloshing, and hydrodynamic coupling between the tube array and the calandria shell. At the same time, the developed method should be general enough to recognize the difference in damping values obtained for isolated tubes in air and in fluid. Such an analytical procedure is presented in this paper to determine the earthquake response of large tube array in partially filled calandria. However, numerical results presented in this paper are limited to harmonic vault excitation.

Whenever the natural frequencies of tubes in an array and the sloshing frequencies of the moderator contained in the calandria are closely spaced, the dynamic response of the tubes is likely to be increased significantly due to tuning effects.[3] However, the larger thickness of the tubes cannot be used to ensure non-vulnerability in a seismic event due to various nuclear and thermodynamic processes inside the calandria that limit the thickness of these tubes.[5] In such a situation, it becomes necessary to control the increase in response of these tubes due to moderator sloshing. For this purpose, the concept of a tuned damper tube in a tube array is proposed in this paper, and its effectiveness has been demonstrated.

SYSTEM AND VAULT MOTION

The system consists of a large horizontal tube array inside a uniform, horizontal calandria (Figure 1). The ends of calandria are welded to the end shields which are grouted in vault walls. The calandria is partially filled with moderator (heavy water) referred in following discussions as inside moderator domain. The fundamental frequency of calandria used in nuclear power plants is usually very high compared with natural frequencies of tubes and the effect of calandria flexibility on the response of tubes is negligible for most engineering applications. Therefore, the calandria and vault walls have been assumed rigid and in a particular direction, harmonic excitation is applied at all points on both supports of calandria.

The analysis procedure has been developed based on the substructure method in frequency domain.[6–8] The reliability of response results depends upon the accuracy in idealization of the substructures. The horizontal fuel tubes have been idealized as one-dimensional flexural beams, and the dynamic equilibrium equations are formulated using admissible continuous functions that satisfy the essential boundary conditions *a priori* (Ritz functions) without any hydrodynamic interaction effects. Approximate homogenization procedure has been used to convert the tube–coolant–fuel rods-spacer system into an equivalent elastic medium with uniformly distributed mass. The total mass of the actual system is preserved in the idealization. The equivalent section modulus of the tube is obtained by matching the analytically evaluated fundamental frequency to experimentally evaluated frequency for the first mode of the tube–coolant–fuel rods-spacer system. The first mode frequency, and the damping value of a single tube–coolant–fuel rods-spacer system in air, with coolant flowing, are regularly evaluated experimentally to identify contact between pressure tubes and coolant tubes.[9] The analytical procedure presented is general enough to include experimentally evaluated mode shapes of the horizontal tubes if they are available. However, one needs to expand them using Ritz functions.

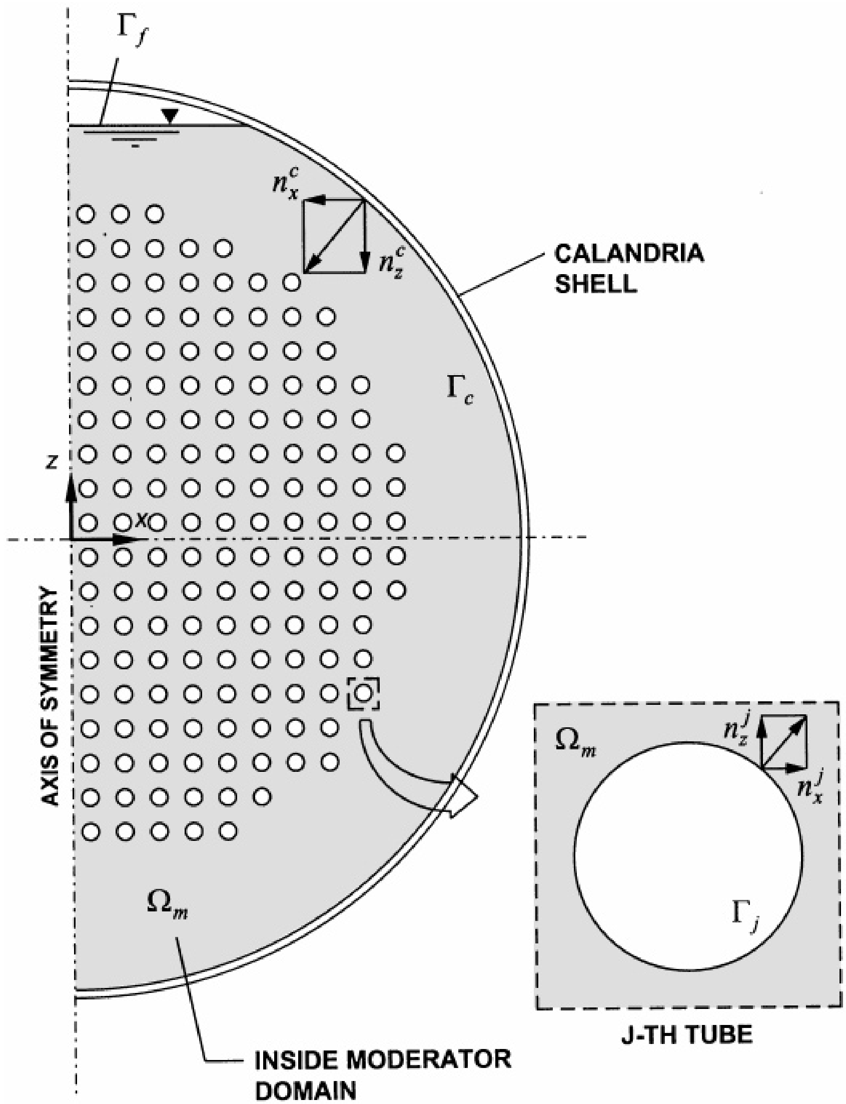

Figure 1. Horizontal tube array in partially filled calandria; domain definitions and normal directions

The frequency response functions of hydrodynamic pressures because of moderator on the tubes and calandria shell are determined by solving the wave equation for the three-dimensional compressible fluid for the inside moderator domain subject to continuity boundary conditions at the structure–fluid interface and the gravity wave condition at the free surface.[10] Damping associated with vibration of free surface has been included by using appropriate damping factor in the gravity wave condition at the free surface.[11] Solutions of the boundary value problems for fluid domain have been obtained by solving three-dimensional problem using semi-analytical process and two-dimensional fluid elements with hydrodynamic pressures as unknowns at nodal

points. The difference in the damping values of tubes in air and fluid has been included in the analysis by introducing the concept of surface damping coefficients for each tube.[7] In the suggested procedure, it is sufficient to establish the surface damping coefficient experimentally for an isolated tube. The dynamic response of the tubes is determined first for the harmonic ground motions at discrete frequencies covering the entire range of interest, and the power spectral density functions of the desired response quantities are then generated using standard techniques.

FREQUENCY DOMAIN EQUATIONS

The governing equations of motion for the tube array including the effects of hydrodynamic interaction are conveniently written in the Fourier-transformed frequency domain because the hydrodynamic pressures depend on the excitation frequency. The idealized system consists of two main substructures: horizontal tubes and the moderator domain. The governing equations for these substructures are presented next, for transverse and vertical components of vault excitation in the frequency domain followed by a general analytical procedure based on the substructure method.

Tube substructure(*s*)

Two independent components of displacement of the jth tube axis are approximated using two sets of continuous functions, one symmetric and second anti-symmetric about $y = H/2$. These functions satisfy the essential boundary conditions for clamped ends and can be written as

$$\bar{u}^j(y,\omega) \approx \sum_{m=2,4,\ldots}^{N_m} \bar{A}_m^{xj}(\omega)\,[1-\cos(\alpha_m y)] + \sum_{m=3,5,\ldots}^{N_m} \bar{A}_m^{xj}(\omega)[\cos(\alpha_1 y)-\cos(\alpha_m y)] \tag{1a}$$

$$\bar{w}^j(y,\omega) \approx \sum_{m=2,4,\ldots}^{N_m} \bar{A}_m^{zj}(\omega)[1-\cos(\alpha_m y)] + \sum_{m=3,5,\ldots}^{N_m} \bar{A}_m^{zj}(\omega)[\cos(\alpha_1 y)-\cos(\alpha_m y)] \tag{1b}$$

where $\alpha_m = m\pi/H$ with H being the length of the tubes and a total of $N_m - 1$ terms are used in the approximation. In equation (1), $\bar{u}^j(y,\omega)$ and $\bar{w}^j(y,\omega)$ are the frequency response functions for the jth tube axis displacements along x- and z-axis, respectively, and $\bar{A}_m^{xj}(\omega)$ and $\bar{A}_m^{jz}(\omega)$ are the frequency response functions of the generalized co-ordinates associated with mth Ritz functions representing tube axis displacements along x- and z-axis. For excitation along x (transverse) and z (vertical) axis, symmetric modes are excited, whereas for excitation along the y (longitudinal) axis, antisymmetric modes are excited. In terms of these generalized co-ordinates, the mass matrix M_j and stiffness matrix K_j of jth tube is generated using the principle of virtual work.[12] In developing the stiffness matrix, only bending deformations are considered and in-plane distortions of the cross-sections have been neglected. The natural frequencies and mode shapes of jth tube without hydrodynamic interaction effects are obtained by solutions of the following associated eigenvalue problem:

$$K_j \Phi_n^j = [\omega_n^j]^2 M_j \Phi_n^j \tag{2}$$

The vector Φ_n^j as given by solution of equation (2) contains coefficients of harmonic functions given in equation (1). These coefficients are rearranged to give the displacements of jth tube in its

nth mode shape in the following form:

$$\bar{\phi}_n^{xj}(y) \approx \sum_{m=0}^{N_m} E_{nm}^{xj} \cos(\alpha_m y) \tag{3a}$$

$$\bar{\phi}_n^{zj}(y) \approx \sum_{m=0}^{N_m} E_{nm}^{zj} \cos(\alpha_m y) \tag{3b}$$

In equations (3a) and (3b), $\sum_{m=0}^{N_m} E_{nm}^{xj} = \sum_{m=0}^{N_m} E_{nm}^{zj} = 0$. In case experimentally evaluated mode shapes for the tubes in air are available, the coefficients E_{nm}^{xj} and E_{nm}^{zj} may be obtained directly by expanding the experimentally evaluated mode shapes in the above form. Since hydrodynamic effects are not yet included, it is sufficient to consider planar vibrations of the tube and obtain mode shapes separately for x and z directions.

The equations of motion in modal co-ordinates, $\bar{Y}_n^j(\omega)$, $n = 1, \ldots, N_j$, for jth tube subject to harmonic excitation $a_x(t) = r_x e^{i\omega t}$ of the vault along the transverse direction, $a_y(t) = r_y e^{i\omega t}$ of the vault along the longitudinal direction and $a_z(t) = r_z e^{i\omega t}$ along the vertical direction are written in frequency domain as[6–8]

$$M_n^j[-\omega^2 + (1 + i\eta_j)[\omega_n^j]^2]\bar{Y}_n^j(\omega) = -L_n^{xj} r_x - L_n^{yj} r_y - L_n^{zj} r_z - \bar{l}_n^j(\omega) \tag{4}$$

in which the generalized mass term M_n^j is given by

$$M_n^j = \int_0^H m^j[\{\phi_n^{xj}(y)\}^2 + \{\phi_n^{zj}(y)\}^2]\, \mathrm{d}y \tag{5}$$

where m^j is the equivalent mass per unit length of the tube after appropriate homogenization procedure, and H is the length of the tube that has been taken same for all tubes in an array. In equation (4), ω_n^j is the nth natural frequency, and η_j is the constant hysteretic damping factor of jth tube in air that needs be established using experimental techniques.[9] Similarly, the generalized excitation terms L_n^{xj}, L_n^{yj} and L_n^{zj} in equation (4) are given by

$$L_n^{xj} = \int_0^H m^j \phi_n^{xj}(y)\, \mathrm{d}y \tag{6a}$$

$$L_n^{yj} = 0 \tag{6b}$$

$$L_n^{zj} = \int_0^H m^j \phi_n^{zj}(y)\, \mathrm{d}y \tag{6c}$$

The hydrodynamic term in equation (4) is given by

$$\bar{l}_n^j(\omega) = \int_{\Gamma_j} (n_x^j\, \phi_{nx}^j + n_z^j\, \phi_{nz}^j)\, \bar{p}^j(\mathbf{x})\, \mathrm{d}\Gamma_j \tag{7}$$

in which n_x^j and n_z^j are the direction cosines of the normal at a point $\mathbf{x}$ (defined by the co-ordinate vector) on the outside surface, Γ_j, of the jth horizontal tube (Figure 1) with respect to x- and z- axis, respectively, and $\bar{p}^j(\mathbf{x})$ is the hydrodynamic pressures on outside surface of the jth horizontal tube.

Inside moderator domain substructure

The frequency response functions of unknown hydrodynamic pressures $\bar{p}^j(\mathbf{x})$ on the jth horizontal tube because of vault excitation along x-, y- and z-axis, can be expressed in terms of

accelerations of modal co-ordinates of the horizontal tubes by analysis of the inside moderator domain. The small amplitude, irrotational motion of inviscid compressible moderator fluid is governed by the three-dimensional wave equation:

$$\nabla^2 \bar{p} + \frac{\omega^2}{C^2} \bar{p} = 0 \tag{8}$$

where $\bar{p}(\mathbf{x}, \omega)$ is the frequency response function for hydrodynamic pressure at a point $\mathbf{x}$ in moderator domain. In this equation C is the velocity of sound in moderator. The hydrodynamic pressures in moderator inside the calandria are generated by acceleration of the inside surface of calandria, and acceleration of the outside surfaces of horizontal tubes. The motion of these boundaries is related to the hydrodynamic pressure by the following boundary conditions:

On the calandria–moderator interface, Γ_c,

$$\frac{\partial}{\partial n^c} \bar{p}(\mathbf{x}, \omega) = -\rho[n^c_x r_x + n^c_z r_z] \tag{9}$$

where ρ is the mass density of moderator, n^c represents the direction of normal to the inside surface of calandria; and n^c_x and n^c_z are the direction cosines of normal at a point $\mathbf{x}$ on the inside surface of the calandria with respect to x- and z-axis, respectively (Figure 1).

On the vault walls–moderator interface, Γ_v,

$$\frac{\partial}{\partial n^v} \bar{p}(\mathbf{x}, \omega) = -\rho n^v_y r_y \tag{10}$$

where n^v represents the direction of normal to the inside surface of vault walls; and n^v_y is the direction cosine of normal at a point $\mathbf{x}$ on the inside surface of the vault walls with respect to y-axis. For flat end calandria, $n^v_y = \pm 1$.

On the jth horizontal tube–moderator interface, Γ_j, in an array of N_t tubes:

$$\frac{\partial}{\partial n^j} \bar{p}(\mathbf{x}, \omega) = -\rho a^j_n(\mathbf{x}, \omega) + \mathrm{i}\omega q_j \bar{p}(\mathbf{x}, \omega), \quad j = 1, \ldots, N_t \tag{11}$$

where n^j represents the direction of the normal to the surface; and $a^j_n(\mathbf{x}, \omega)$ is the spatial distribution of the acceleration of the outside surface of jth horizontal tube in its normal direction. The mechanism of added damping due to fluid–structure interaction is not yet well understood. This aspect however has been included in the analysis procedure through the introduction of generalized surface damping coefficient, q_j for the jth tube. This coefficient needs to be established, only for an isolated tube in fluid, by a separate analysis to match the experimentally obtained frequency response functions. A simple procedure is presented in the subsequent section.

On the free surface of the moderator, Γ_f, assuming small free surface waves:

$$(1 + \mathrm{i}\eta_s) \frac{\partial}{\partial z} \bar{p}(\mathbf{x}, \omega) = \frac{\omega^2}{g} \bar{p}(\mathbf{x}, \omega) \tag{12}$$

where η_s is the equivalent damping factor associated with sloshing of free surface.[11]

For harmonic excitation of the calandria vault with components r_x, r_y, and r_z along the transverse, longitudinal and vertical directions, respectively, the surface accelerations $a^j_n(\mathbf{x}, \omega)$ on the jth horizontal tube-moderator interface, Γ_j, are related to the modal co-ordinates $\bar{Y}^j_n(\omega)$, $n = 1, \ldots, N_j$, and associated displacement functions, $\phi^{xj}_n(y)$ and $\phi^{zj}_n(y)$, of the jth tube in the

following way:

$$a_n^j(\mathbf{x},\omega) = n_x^j r_x + n_z^j r_z - \omega^2 \sum_{n=1}^{N_j} [n_x^j \phi_n^{xj}(y) + n_z^j \phi_n^{zj}(y)]\, \bar{Y}_n^j(\omega) \tag{13}$$

The linear form of the governing equation and boundary conditions allow $\bar{p}(\mathbf{x},\omega)$ to be expressed as

$$\bar{p}(\mathbf{x},\omega) = \bar{p}_{0x}(\mathbf{x},\omega) r_x + \bar{p}_{0y}(\mathbf{x},\omega) r_y + \bar{p}_{0z}(\mathbf{x},\omega) r_z - \omega^2 \sum_{j=1}^{N_t} \sum_{n=1}^{N_j} \bar{p}_n^j(\mathbf{x},\omega)\, \bar{Y}_n^j(\omega) \tag{14}$$

In equation (14), the hydrodynamic pressure functions $\bar{p}_{0x}(\mathbf{x},\omega)$, $\bar{p}_{0y}(\mathbf{x},\omega)$ and $\bar{p}_{0z}(\mathbf{x},\omega)$ are the solutions of equation (8) for *a rigid calandria and rigid horizontal tubes* due to unit acceleration of vault along x-, y- and z-axis, respectively. Similarly, in equation (14), the hydrodynamic pressure function $\bar{p}_n^j(\mathbf{x},\omega)$ is the solution of the governing equation, due to excitation of jth horizontal tube in its nth mode with no excitation of the vault, calandria walls or other tubes. The numerical evaluation procedure to solve these boundary value problems is presented in the subsequent section.

Coupled equations of motion

The modal equations of motion [equation (4)] for horizontal tubes in $\sum_j^{N_t} N_j$ unknowns (the frequency response function for the generalized co-ordinates $\bar{Y}_n^j(\omega)$ associated with the first N_j modes of vibration of jth horizontal tube without moderator, $j = 1, \ldots, N_t$) can be written in the following form by substituting equation (14) into equation (4):

$$\begin{aligned} -\omega^2 M_n^j \bar{Y}_n^j(\omega) - \omega^2 \sum_k^{N_t} \sum_l^{N_k} \bar{M}_{nkl}^{ja}(\omega) \bar{Y}_l^k(\omega) + M_n^j[(1+\mathrm{i}\eta_j)[\omega_n^j]^2]\, \bar{Y}_n^j(\omega) \\ = -r_x[L_n^{xj} + \bar{L}_n^{xja}(\omega)] - r_y[L_n^{yj} + \bar{L}_n^{yja}(\omega)] - r_z[L_n^{zj} + \bar{L}_n^{zja}(\omega)],\ n = 1, \ldots, N_j \end{aligned} \tag{15}$$

where N_k is the number of modes considered for k-th tube and

$$\bar{M}_{nkl}^{ja}(\omega) = \int_{\Gamma_j} [n_x^j \phi_n^{xj} + n_z^j \phi_n^{zj}]\, \bar{p}_l^k \, \mathrm{d}\Gamma_j \tag{16}$$

$$\bar{L}_n^{xja}(\omega) = \int_{\Gamma_j} (n_x^j \phi_n^{xj} + n_z^j \phi_n^{zj}) \bar{p}_{0x} \, \mathrm{d}\Gamma_j \tag{17a}$$

$$\bar{L}_n^{yja}(\omega) = \int_{\Gamma_j} (n_x^j \phi_n^{xj} + n_z^j \phi_n^{zj}) \bar{p}_{0y} \, \mathrm{d}\Gamma_j \tag{17b}$$

$$\bar{L}_n^{zja}(\omega) = \int_{\Gamma_j} (n_x^j \phi_n^{xj} + n_z^j \phi_n^{zj}) \bar{p}_{0z} \, \mathrm{d}\Gamma_j \tag{17c}$$

In equation (15), for $j = k$, the real part of the function $\bar{M}_{nkl}^{ja}(\omega)$ represents the self-added hydrodynamic mass on the jth horizontal tube in its nth mode due to its own motion, and the imaginary part of the function $\bar{M}_{nkl}^{ja}(\omega)$ represents the self-added hydrodynamic damping of jth horizontal tube motion in its n-th mode due to its own motion. For $j \neq k$, the real part of the function $\bar{M}_{nkl}^{ja}(\omega)$ represents the coupled-added hydrodynamic mass on the jth horizontal tube in its nth mode due to motion of the kth tube in its lth mode, and the imaginary part of function

$\bar{M}^{ja}_{nkl}(\omega)$ represents the coupled-added hydrodynamic damping of jth horizontal tube motion in its nth mode due to motion of the k-th tube in its lth mode. In equation (15), the terms $\bar{L}^{xja}_{n}(\omega)$, $\bar{L}^{yja}_{n}(\omega)$, and $\bar{L}^{zja}_{n}(\omega)$ are the added hydrodynamic excitations in the nth mode of vibration of jth horizontal tube due to vault excitation along x-, y- and z-axis, respectively.

Thus there are a total of $\sum_{j}^{N_t} N_j$ frequency-dependent equations that are coupled through the hydrodynamic forces. These equations must be solved simultaneously for each excitation frequency of interest to get the frequency response functions $\bar{Y}^{j}_{n}(\omega)$ (the generalized modal co-ordinate associated with the first N_j modes of vibration of jth horizontal tube). Repeated solution for the excitation frequencies covering the range over which the vault excitations and structural response have significant components leads to the complete frequency response functions for the modal co-ordinates.

Response to specified vault excitation

The frequency response function of any response quantity for j-th horizontal tube, $R_j(\omega)$ can be expressed in terms of the frequency response functions for modal co-ordinates $\bar{Y}^{j}_{n}(\omega)$ of jth horizontal tube:

$$R_j(\omega) = \sum_{n=1}^{N_j} R^j_n \bar{Y}^j_n(\omega) \tag{18}$$

where the coefficients R^j_n define the desired response quantity in nth mode for the jth horizontal tube under consideration. The frequency response function of any response quantity $R_j(\omega)$, for jth tube, is computed for three sets of values of r_x, r_y and r_z, thus yielding $R_{jx}(\omega)$ for $r_x = 1$, $r_y = 0$, and $r_z = 0$, $R_{jy}(\omega)$ for $r_x = 0$, $r_y = 1$, and $r_z = 0$, and $R_{jz}(\omega)$ for $r_x = 0$, $r_y = 0$, and $r_z = 1$. The power spectral density of response $S_{RR}(\omega)$ of response quantity $R(t)$, for jth tube, is given by

$$S_{RR}(\omega) = \begin{Bmatrix} \bar{R}_{jw}(\omega) \\ \bar{R}_{jy}(\omega) \\ \bar{R}_{jz}(\omega) \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} S^{\mathrm{v}}_{xx}(\omega) & S^{\mathrm{v}}_{xy}(\omega) & S^{\mathrm{v}}_{xz}(\omega) \\ S^{\mathrm{v}}_{yx}(\omega) & S^{\mathrm{v}}_{yy}(\omega) & S^{\mathrm{v}}_{yz}(\omega) \\ S^{\mathrm{v}}_{zx}(\omega) & S^{\mathrm{v}}_{zy}(\omega) & S^{\mathrm{v}}_{zz}(\omega) \end{bmatrix} \begin{Bmatrix} \bar{R}_{jw}(\omega) \\ \bar{R}_{jy}(\omega) \\ \bar{R}_{jz}(\omega) \end{Bmatrix} \tag{19}$$

where $S^{\mathrm{v}}_{xx}(\omega)$, $S^{\mathrm{v}}_{xy}(\omega)$, $S^{\mathrm{v}}_{xz}(\omega)$, $S^{\mathrm{v}}_{yx}(\omega)$, $S^{\mathrm{v}}_{yy}(\omega)$, $S^{\mathrm{v}}_{yz}(\omega)$, $S^{\mathrm{v}}_{zx}(\omega)$, $S^{\mathrm{v}}_{zy}(\omega)$ and $S^{\mathrm{v}}_{zz}(\omega)$ are the terms of power spectral density function matrix for vault excitations.

NUMERICAL EVALUATION PROCEDURES

Hydrodynamic pressures

The geometry of inside moderator domain does not change along the y-axis. This allows surface accelerations and unknown pressure to be expanded as Fourier series and solution for each term of the series separately utilizing the orthogonality property of trigonometric functions. For pressure functions $\bar{p}_{0x}(\mathbf{x}, \omega)$, $\bar{p}_{0y}(\mathbf{x}, \omega)$, $\bar{p}_{0z}(\mathbf{x}, \omega)$, and $\bar{p}^{j}_{n}(\mathbf{x}, \omega)$, $n = 1, \ldots, N_j$; $j = 1, \ldots, N_t$, each of the three-dimensional boundary value problem is reduced to a number of two-dimensional

boundary value problems by expanding the pressure functions in the following form:

$$\bar{p}_{0x}(\mathbf{x},\omega) = \sum_{m=0,2,\ldots}^{N_m} \bar{p}_{0xm}(x,z,\omega)\cos(\alpha_m y) \tag{20a}$$

$$\bar{p}_{0y}(\mathbf{x},\omega) = \sum_{m=1,3,\ldots}^{N_m} \{\beta_m + \bar{p}_{0ym}(x,z,\omega)\cos(\alpha_m y) \tag{20b}$$

$$\bar{p}_{0z}(\mathbf{x},\omega) = \sum_{m=0,2,\ldots}^{N_m} \bar{p}_{0zm}(x,z,\omega)\cos(\alpha_m y) \tag{20c}$$

$$\bar{p}^j_n(\mathbf{x},\omega) = \sum_{m=0}^{N_m} \bar{p}^j_{xm}(x,z,\omega)\cos(\alpha_m y) \tag{20d}$$

These pressure functions are obtained by solving the following two-dimensional equation repeatedly in x–z plane for each m:

$$\nabla^2\bar{p}_m + \left[\frac{\omega^2}{C^2} - \alpha_m^2\right]\bar{p}_m = 0 \tag{21}$$

In equation (20b) β_m is given as

$$\beta_m = \frac{4\rho r_y}{\alpha_m^2 H(1-\omega^2/\alpha_m^2 C^2)} \tag{22}$$

and this expansion satisfies the governing wave equation as well as the boundary condition at Γ_v.

However, in equations (20a) and (20c), only one term corresponding to $m = 0$ is non-zero as the surface accelerations are constant along the y-axis. Therefore, the pressure function $\bar{p}_{0x}(\mathbf{x},\omega)$ is obtained by solving two-dimensional version of equation (8) in the x–z plane, that is equation (21) with $\alpha_m = m\pi/H = 0$, for the following boundary conditions:

$$\frac{\partial}{\partial n^c}\,\bar{p}_{0x}(\mathbf{x},\omega) = -\,\rho n_x^c(x,z) \quad \text{on } \Gamma_c \tag{23a}$$

$$\frac{\partial}{\partial n^j}\,\bar{p}_{0x}(\mathbf{x},\omega) = [-\rho n_x^j(x,z) + \mathrm{i}\omega q_j \bar{p}_{0x}(\mathbf{x},\omega)] \quad \text{on } \Gamma_j \tag{23b}$$

$$(1+\mathrm{i}\eta_s)\,\frac{\partial}{\partial z}\bar{p}_{0x}(\mathbf{x},\omega) = \frac{\omega^2}{g}\,\bar{p}_{0x}(\mathbf{x},\omega) \quad \text{on } \Gamma_f \tag{23c}$$

Similarly, the solution for pressure function $\bar{p}_{0z}(\mathbf{x},\omega)$ is obtained by solving two-dimensional version of equation (8) in the x–z plane for the boundary conditions of equation (23) with n_x^c and n_x^j replaced by n_z^c and n_z^j, respectively, in equations (23a) and (23b).

For pressure function $\bar{p}_{0y}(\mathbf{x},\omega)$, the solution for pressure functions $\bar{p}_{0ym}(x,z,\omega)$ is obtained by solving the two-dimensional equation (21) repeatedly in the x–z plane for $m = 1, 3, \ldots, N_m$ subject to the following boundary conditions:

$$\frac{\partial}{\partial n^c}\,\bar{p}_{0ym}(x,z,\omega) = 0 \quad \text{on } \Gamma_c \tag{24a}$$

$$\frac{\partial}{\partial n^j}\,\bar{p}_{0ym}(x,z,\omega) = \mathrm{i}\omega q_j \bar{p}_{0ym}(\mathbf{x},z,\omega) \quad \text{on } \Gamma_j \tag{24b}$$

$$(1+\mathrm{i}\eta_s)\frac{\partial}{\partial z}\bar{p}_{0ym}(x,z,\omega) = \frac{\omega^2}{g}\,\{\bar{p}_{0ym}(x,z,\omega)+\beta_m\} \quad \text{on } \Gamma_{\mathrm{f}} \tag{24c}$$

For pressure functions $\bar{p}^j_n(\mathbf{x},\omega)$, $n = 1, \dots, N_j$; $j = 1, \dots, N_t$, the solution for pressure functions $\bar{p}^j_{nm}(x,z,\omega)$ is obtained by solving the two-dimensional equation (21) repeatedly in x–z plane for each m and subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{\mathrm{c}}}\,\bar{p}^j_{nm}(x,z,\omega) = 0 \quad \text{on } \Gamma_{\mathrm{c}} \tag{25a}$$

$$\frac{\partial}{\partial n^j}\bar{p}^j_{nm}(x,z,\omega) = -\,\rho[n^j_x E^{xj}_{nm} + n^j_z E^{zj}_{nm}] + \mathrm{i}\omega q_j \bar{p}^j_{nm}(x,z,\omega) \quad \text{on } \Gamma_j \tag{25b}$$

$$(1+\mathrm{i}\eta_s)\frac{\partial}{\partial z}\,\bar{p}^j_{nm}(x,z,\omega) = \frac{\omega^2}{g}\,\bar{p}^j_{nm}(x,z,\omega) \quad \text{on } \Gamma_{\mathrm{f}} \tag{25c}$$

These boundary conditions are derived by substituting equation (13) into equations (9)-(12), and then expanding the right-hand side of equations (9)–(12) using the analytical functions $\cos(\alpha_m y)$. The pressure functions used on the right-hand side of equation (14) are then obtained using the summation for all m [equation (20)].

These boundary value problems are solved by minimization of the following functional:[13]

$$\Pi = \frac{1}{2}\int_{\Omega_m} \nabla p \cdot \nabla p \, \mathrm{d}\Omega - \frac{1}{2}\left[\frac{\omega^2}{C^2} - (\alpha_m)^2\right]\int_{\Omega_m} (\bar{p})^2 \, \mathrm{d}\Omega - \frac{\omega^2}{2g(1+\mathrm{i}\eta_s)}\int_{\Gamma_f} (\bar{p})^2 \, \mathrm{d}\Gamma$$
$$-\,\rho\int_{\Gamma_{\mathrm{c}}} b^{\mathrm{c}}\bar{p}\,\mathrm{d}\Gamma - \rho\sum_j \int_{\Gamma_j} b^j \bar{p}\,\mathrm{d}\Gamma + \sum_j \mathrm{i}\omega q_j \int_{\Gamma_j} (\bar{p})^2\,\mathrm{d}\Gamma - \frac{\omega^2\beta_m}{g(1+\mathrm{i}\eta_s)}\int_{\Gamma_f} \bar{p}\,\mathrm{d}\Gamma \tag{26}$$

using standard finite-element procedures.[14] The last term is required for considering excitation along the y-axis, that is evaluation of pressure function $\bar{p}_{0y}(\mathbf{x},\omega)$. In equation (26), b^{c} and b^j are the known functions defining distribution of surface accelerations at calandria–moderator interface and the jth tube–moderator interface, respectively. The moderator domain Ω_m is idealized as an assemblage of two-dimensional finite elements and consequently, the surfaces Γ_{c}, Γ_j, and Γ_{f} get discretized into a number of subdivisions (Figure 2). At each frequency interval, $2 + 2N_t$ solutions of simultaneous complex equations derived from equation (26) are required for $m = 0$ while for $m = 1, \dots, N_m$, the simultaneous equations are solved $2N_t$ times if m is even and $1 + 2N_t$ times if m is odd. However, to generate frequency response functions for hydrodynamic pressures, the solutions of simultaneous equation derived from equation (26) are repeated for each frequency value to cover the entire range of interest.

The pressure values obtained by solutions of equation (26) are used in equations (16) and (17) for the evaluation of complex-valued added mass, and added excitation terms. The integration over three-dimensional surface required in these equations is carried out using Lagrange interpolation of the nodal pressure values and Gaussian numerical integration in x–z plane while integration along the y-axis is performed explicitly.

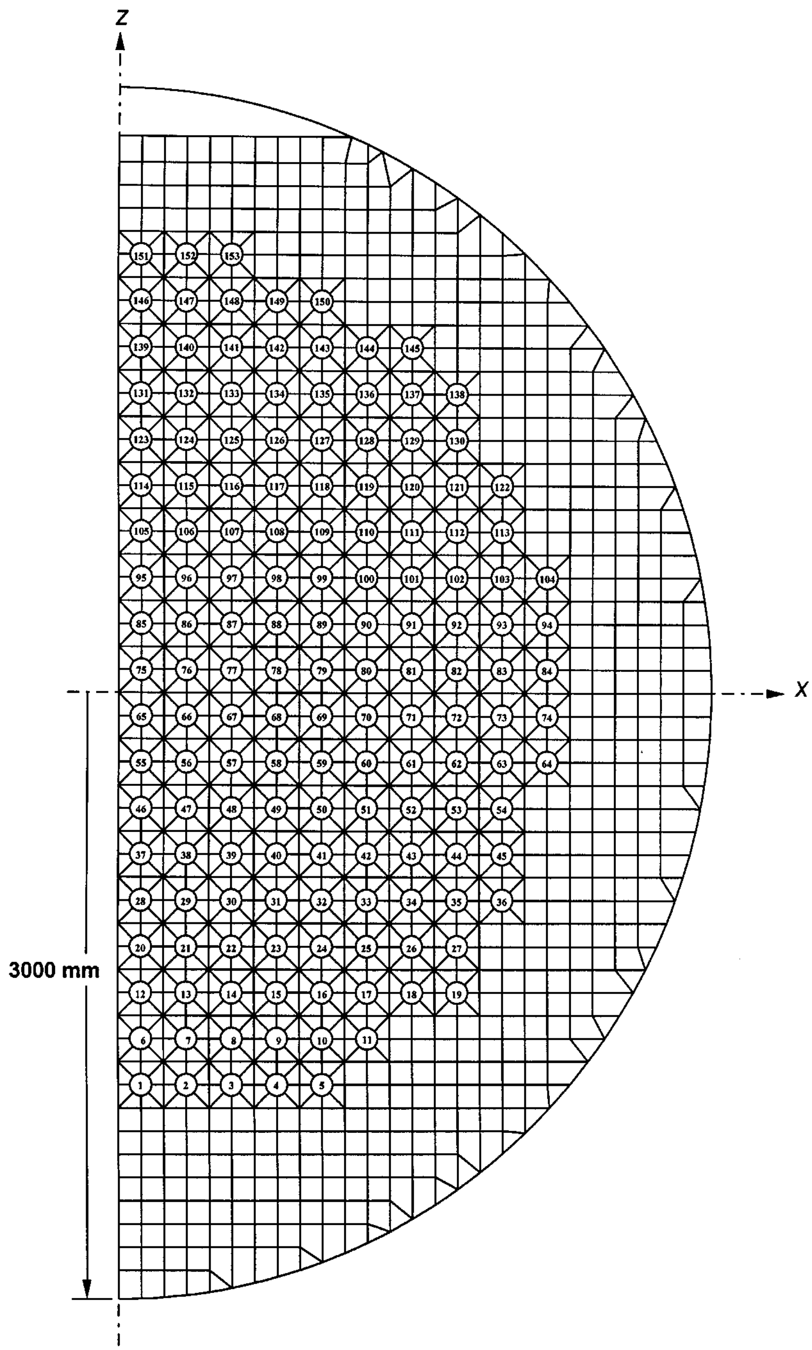

Figure 2. Finite element idealization of inside moderator domain

Equivalent surface damping

The experimental evidence of added damping for submerged tubes is well documented in literature,[4,15] though the mechanism of added damping due to fluid–structure interaction is not yet well understood. A simple procedure is proposed to include this aspect in the analysis through the introduction of a generalized surface damping coefficient, q_j for the jth tube, characterizing the effects of absorption of hydrodynamic pressure waves at tube surface. In substructure method, the reliability of response results depends upon the accuracy in idealization of the substructures. Therefore, this coefficient may be established from the results of an experiment conducted using a single tube.

In the proposed procedure, the dynamic properties of the actual tube assembly in air including equivalent hysteretic damping coefficient need be established first. The frequency response function for the same tube is then measured experimentally in a rectangular tank filled with moderator. The size of the rectangular tank is chosen to have the first sloshing frequency equal to the transverse sloshing frequency of actual calandria. The measured frequency response function of the tube is then compared with the frequency response functions evaluated analytically using the presented procedure with different values of surface damping coefficient, q_j along with the hysteretic damping coefficient for the tube assembly in air. The value of surface damping coefficient for which the analytical frequency response function associated with the first mode matches with the experimentally measured function, should then be used for the coupled analysis of the complete tube array in actual calandria.

The effects of the introduction of surface damping coefficient on the response have been demonstrated for a 5 m long tube (outside diameter of 110.2 mm, fundamental frequency 7.7 Hz and hysteretic damping coefficient in air equal to 0.04) placed horizontally in a rectangular tank. The finite element system used in the analysis is shown in Figure 3. The frequency response functions for the generalized co-ordinate associated with the first mode are also presented in Figure 3 for three different values of surface damping coefficients. Three modes of the tube have been considered for tank excitation along the x-axis. These response results demonstrate that added hydrodynamic damping effects can be simulated in analysis through surface damping coefficients. The response results, however, are sensitive to the generalized surface damping coefficient assumed, and therefore, these coefficients must be evaluated experimentally at least for an isolated tube as the experimental evaluation of entire tube array may not be feasible.

RESPONSE RESULTS

The analytical procedure and numerical evaluation techniques presented earlier for the evaluation of the added hydrodynamic mass, damping and excitation terms in the coupled equations of motion have been implemented in a set of computer programs. The analysis procedure has been utilized to develop a fundamental understanding of how the hydrodynamic coupling effects and moderator sloshing influence the dynamic response of tube array in calandria. To this end the response of tubes in calandria to harmonic vault excitation varied over a wide range of excitation frequencies have been determined.

The numerical results are presented for a typical calandria (length 5000 mm and inside radius 3000 mm) used in 235 MW nuclear power plants of India. Homogenized tube model of actual tube assembly with uniform mass per unit length having fundamental frequency in air equal to

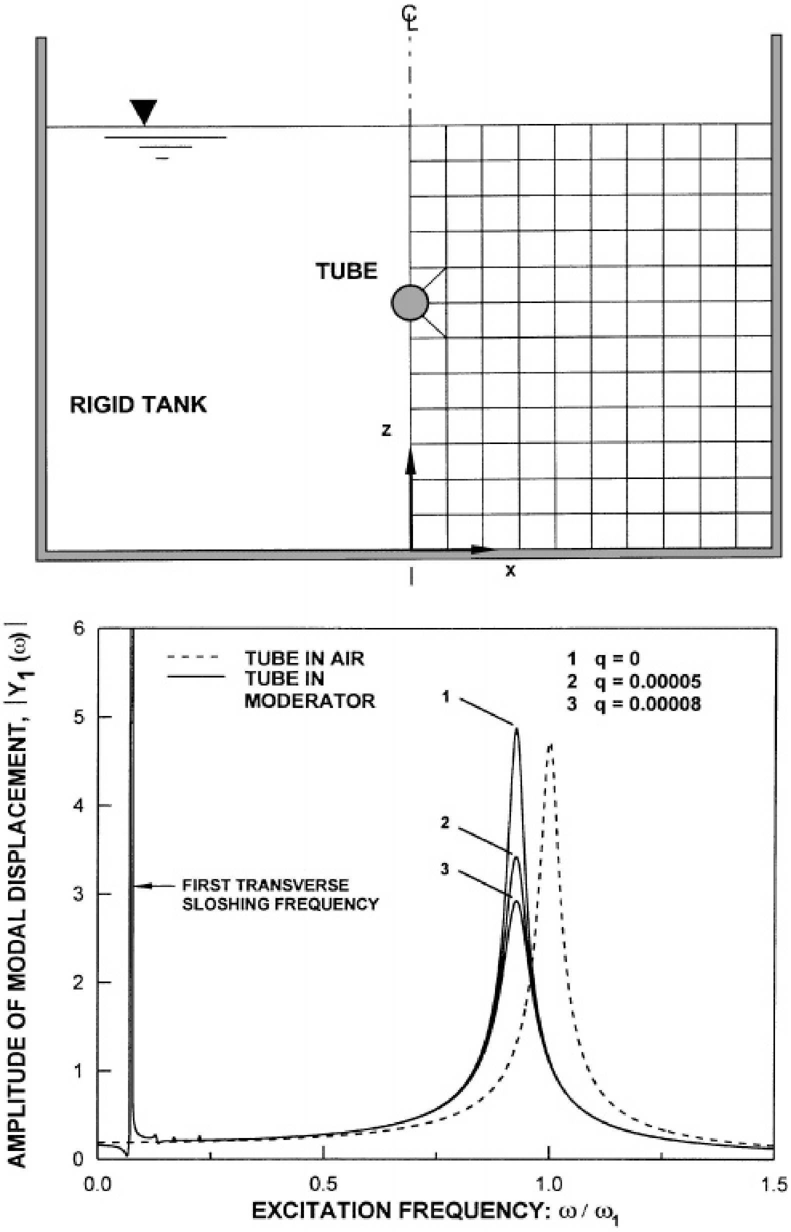

Figure 3. Finite element system and response results for an isolated tube in a rectangular tank with different surface damping coefficients

7·7 Hz and a hysteretic damping coefficient of 0·04 (2 per cent damping) have been used. The Moderator is assumed to be compressible, inviscid liquid with mass density and bulk modulus equal to that of water. The depth of moderator at the vertical centreline of calandria is 96 per cent of its inside diameter. The Vault is assumed rigid and its translational motions, along x-, y- and z-axis, have been considered.

For analysis of fluid domain using semi-analytical process, different boundary conditions have been imposed on the line of symmetry for excitation along x-, y- and z-axis. For vault excitation along the x- and z-axis, six symmetric modes of each tube (three in each x and z directions) have been considered simultaneously, whereas for vault excitation along the y-axis, three anti-symmetric modes in each x and z directions have been considered. Therefore, analysis has been performed separately for vault excitations along x-, y- and z-axis. Twelve terms have been used in the semianalytical process. In presenting numerical results, the sloshing modes have been assumed undamped. The response has been found for the frequency range of 0–25 Hz. The finite element discretization of fluid domain is fine enough to have contribution of first 20 sloshing modes to the tube response.

For vault excitation along the y-axis sloshing of moderator excites antisymmetric modes. The natural frequency corresponding to the first anti-symmetric mode of the tubes (21·09 Hz) is far apart from the first longitudinal sloshing frequency (0·44 Hz) and is far beyond the frequency range in which earthquake motions have significant energy. All this leads to a negligible response of the tube array to vault excitation along the y-axis. Figure 4 shows response of two tubes located in the top and bottom rows of the array to vault excitation along x-, y- and z-axis, respectively. The contribution of the y-axis excitation to the response, even for a tube close to the free surface, is negligible compared with that due to excitation along the x- and z-axis. Therefore, in further investigations, efforts are concentrated on understanding the behaviour of the system subject to vault excitation along vertical and transverse directions only.

Hydrodynamic coupling effects

In order to quantify the coupling effects, two-dimensional analyses with only rigid body motions of calandria and tubes are performed using potential theory. The net hydrodynamic force on four tubes, identified by numbers 153, 84, 75 and 3 (Figure 2), and located at different sections of calandria, are presented in Figure 5 for both the directions of excitation, separately. The forces have been normalized to the inertial force due to unit acceleration of the displaced mass of moderator. The response results are presented for four cases: (1) for unit acceleration of the tube under consideration with other tubes and calandria stationary, (2) for unit acceleration of all other tubes except that of the tube under consideration with calandria stationary, (3) for unit acceleration of the calandria with all tubes stationary, and (4) for simultaneous unit acceleration of calandria and all the tubes, a combination of cases (1) to (3). These forces are equivalent to hydrodynamic added mass in conventional analysis.

Two types of coupling effects are present in such a complex system: coupling between the tubes and calandria shell, and coupling between various tubes. The hydrodynamic forces on a particular tube are also influenced by the motion of the relatively large and stiff calandria, and the motion of calandria exerts forces that are not in the direction of applied excitation. The hydrodynamic forces on a particular tube are also influenced by the motion of the tubes surrounding it, though the influence is significant only on cross response, that is response along the z-axis for excitation along the x-axis. This implies that the conventional added mass approach

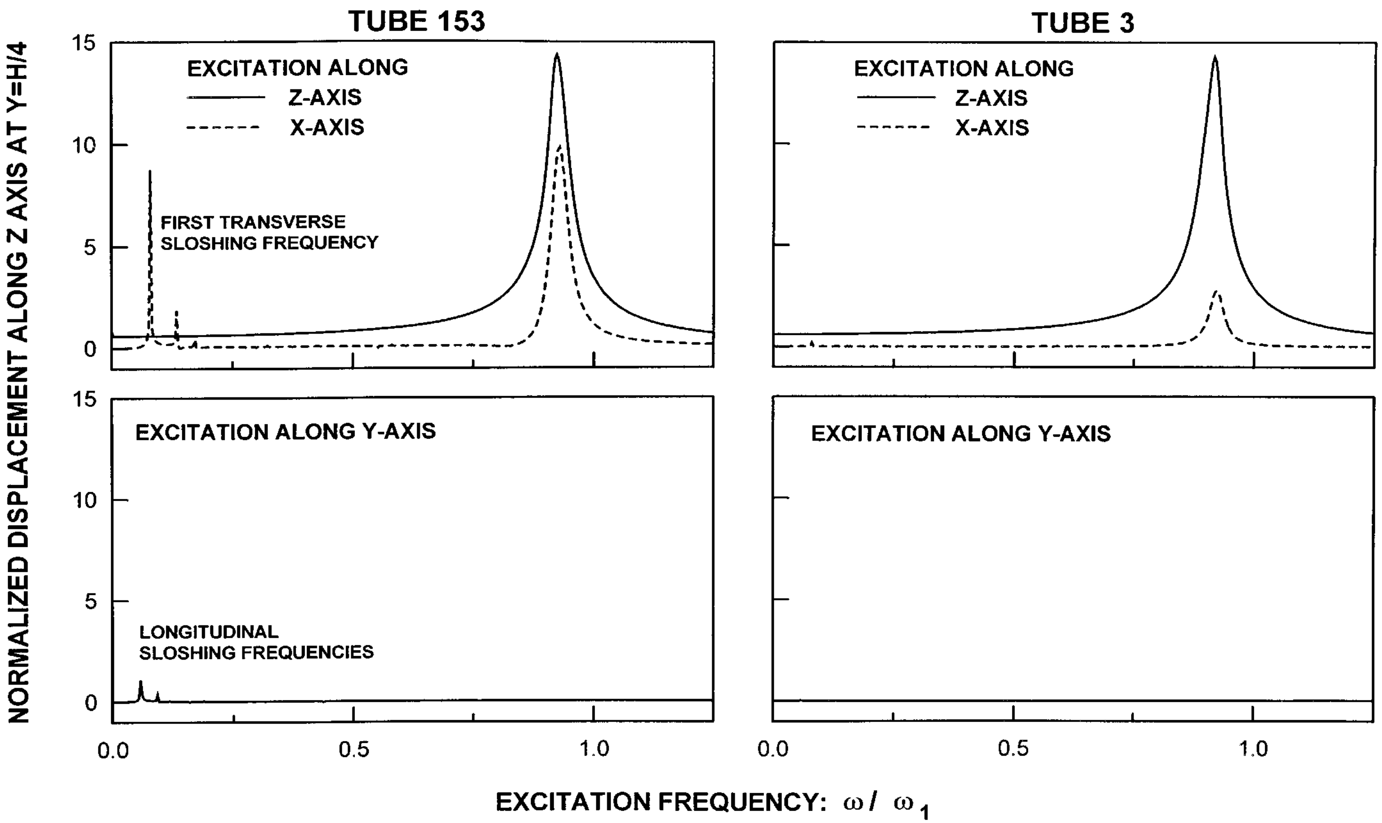

Figure 4. Response of tubes to vault excitation along *x*-, *y*- and *z*-axis

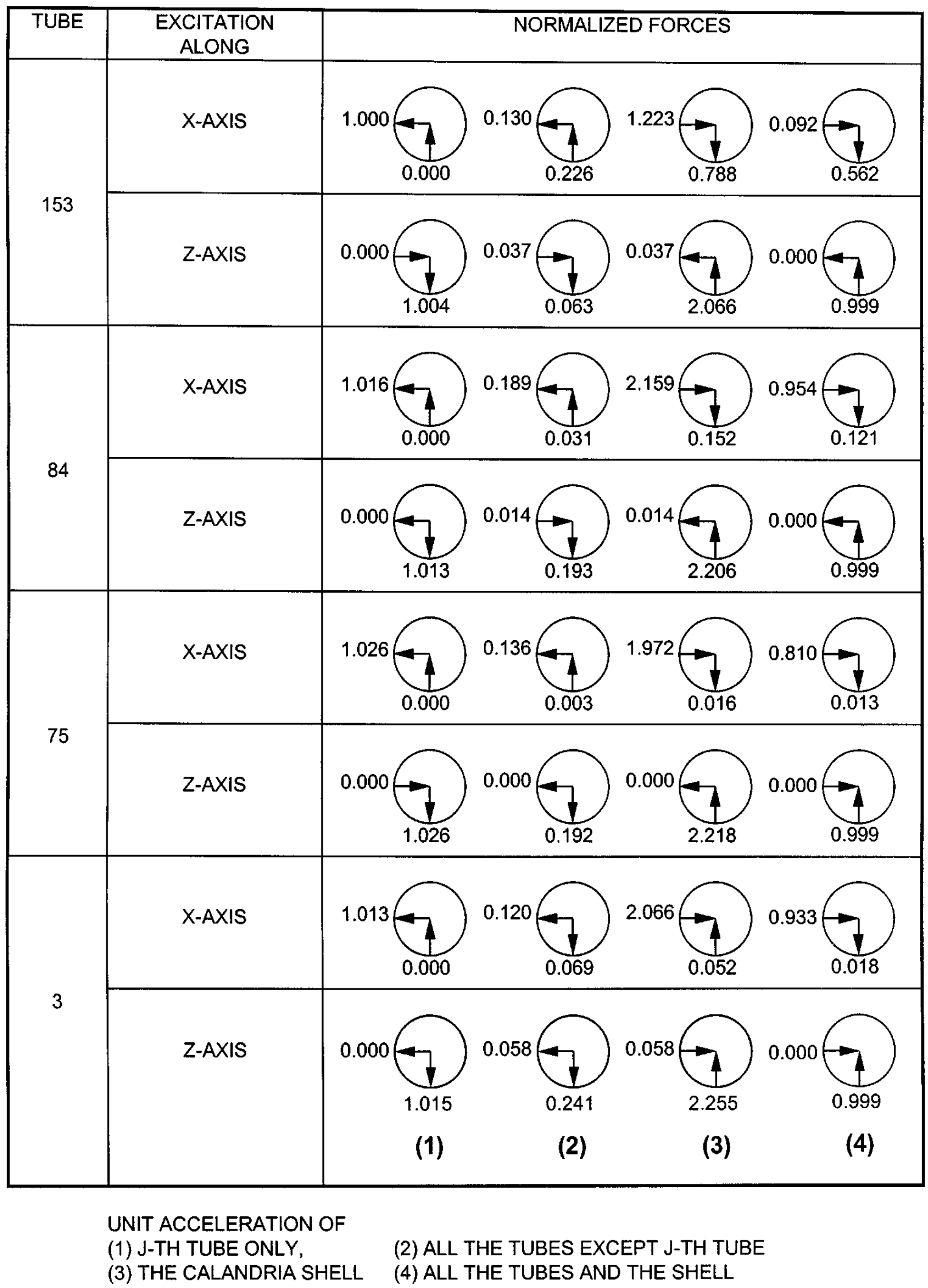

Figure 5. Hydrodynamic forces on tubes

applied to individual tubes may not be appropriate because it does not take into account the coupling effects. Additionally, the assumption that all the tubes and calandria move in phase and with same amplitude, that is necessary for conventional added mass approach applied to the entire system, is also not valid.

To quantify the contribution to response because of the motion of other tubes, the frequency response functions for mid-span displacements are presented in Figure 6 for two tubes 153 and 3, located in different regions of the tube array. The response of these tubes along both the x- and z-axis due to excitation separately along the x- and z-axis is presented. The responses of these tubes, considered one at a time in a similar system with other tubes removed, are also presented. As shown in Figure 6, one-tube model is able to represent responses in the direction of excitation fairly accurately however cross response results are grossly in error near natural frequency of tubes. This is true for excitation along either direction. Tube 153 is near calandria shell and has maximum cross-response due to the vicinity of free surface. The tubes do not vibrate in phase and with the same amplitude due to hydrodynamic effects, and therefore, flexibility of all the tubes needs be considered along both the directions simultaneously to get correct response.

Sloshing effects

For the system analysed, the ratio of tube frequency in air to transverse sloshing frequency of moderator in calandria (frequency ratio) is 12·6. Hence, the two fundamental frequencies are far apart, and consequently, the tuning is almost non-existent.[16] The calandria is much more stiff compared to the tubes, and therefore has been assumed rigid in this analysis. The sloshing is confined only to a narrow frequency range and to the top few rows of the tubes (Figure 6). The sloshing forces acting in x and z directions due to excitation in x direction are almost of the same order. However, sloshing effects diminish fast as one goes deeper. Additionally, the effects of sloshing can be neglected for z-axis excitation without introducing significant errors.

To investigate the effects of frequency tuning on the response of tubes, the frequency of the tubes is adjusted numerically so that frequency ratio is two. The direct response of typical tubes due to excitation along the x-axis are presented in Figure 7 for two frequency ratios (12·6 and 2). The response results are presented only for the tubes that are closer to the free surface and are most influenced by sloshing. As shown in this figure, the response is influenced much more by the sloshing effects when the natural frequencies are closer. If the designed system has tuning of frequencies, one must control the sloshing effects or design for significantly higher hydrodynamic forces. However, it may not be possible to increase the thickness of the tube due to associated problems with increased thickness in various nuclear and thermodynamic processes.[5]

To overcome this difficulty, the concept of a tuned damper tube has been investigated. It is proposed that for one of the tubes, most influenced by sloshing, natural frequency should be adjusted to match the first sloshing frequency, and the tuned tube should have additional surface damping. This concept has been used for this calandria to explore the possibility of using to advantage the frequency tuning of one tube to bring down the contribution of sloshing to response of other tubes.

To demonstrate the effects of tuning of one tube, the numerical results are presented for two cases. In the first case, the frequency of tube 153 in air is set equal to that of the first transverse sloshing frequency, while for rest of the tubes the frequency ratio is kept at two; in the second case, the frequency ratio is two for all the tubes. Small surface damping is also introduced in the tuned tube (tube 153) to increase its damping from 2 per cent in air to 3 per cent in moderator in the first

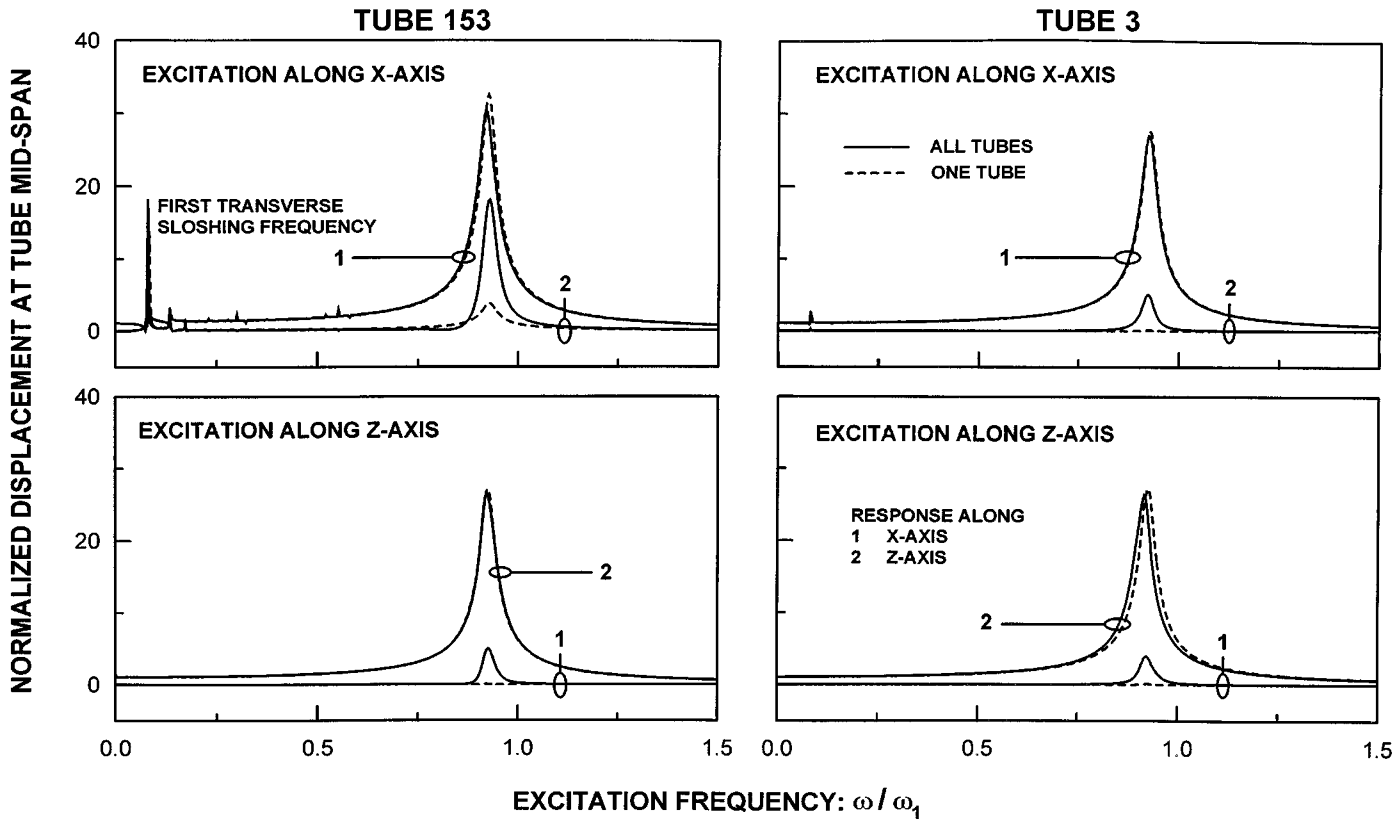

Figure 6. Effects of hydrodynamic coupling on direct and cross responses of tubes

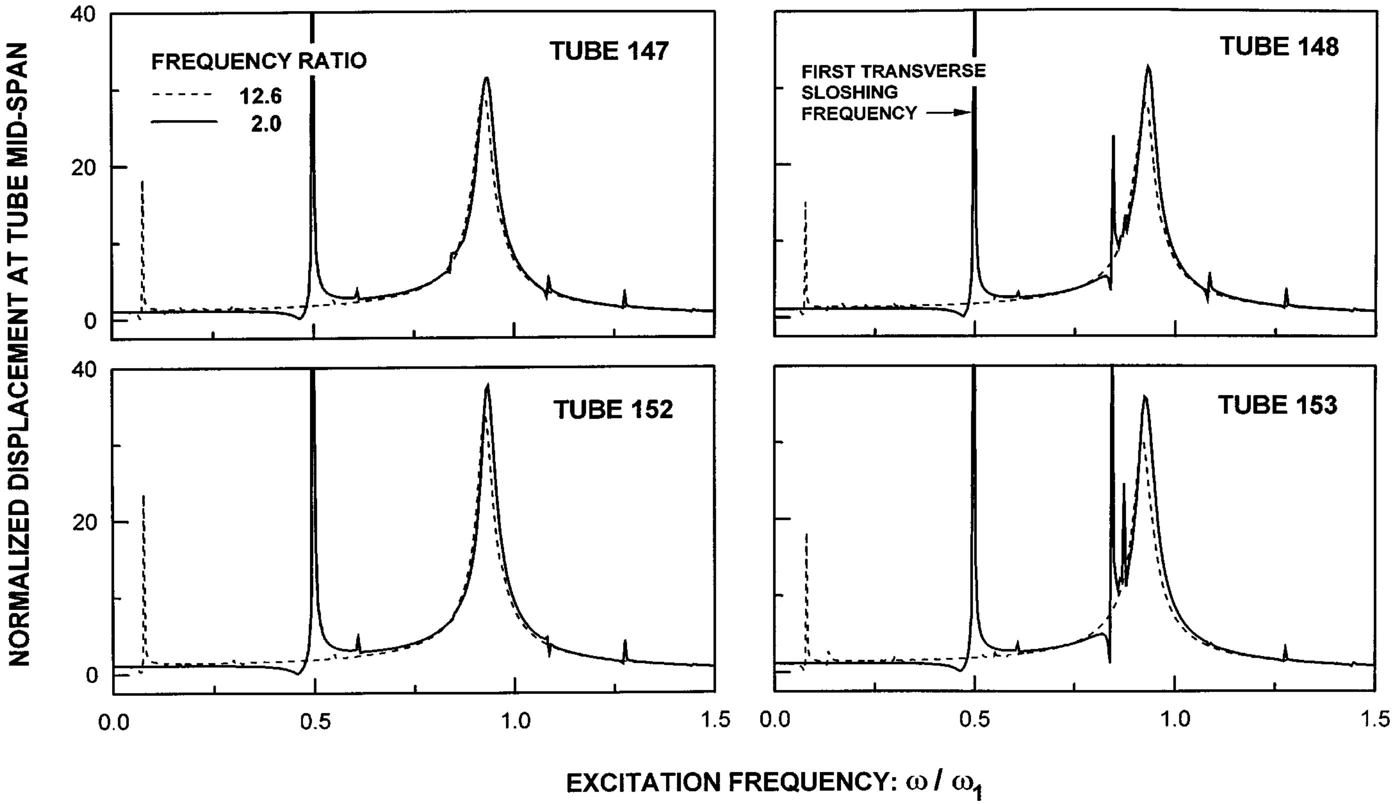

Figure 7. Effects of tuning of tube and sloshing frequencies on responses of tubes

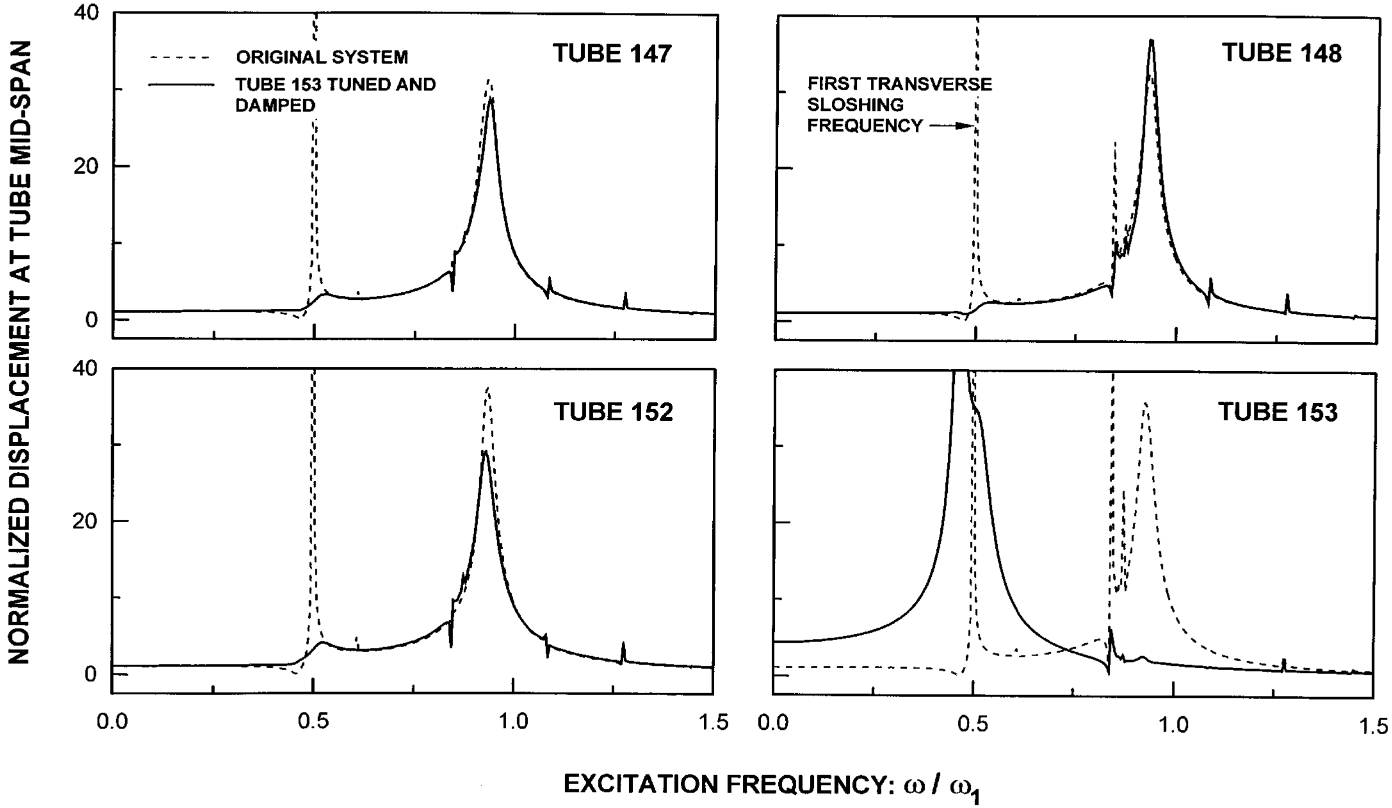

Figure 8. Effects of tuned damper tube (tube 153) on responses of neighbouring tubes

case. The frequency response functions for mid-span displacements of tubes 147, 148, and 152, surrounding the tuned damper tube 153 are presented in Figure 8 for the two cases. It is observed that near the peak corresponding to tube frequency the response is not much influenced. However, the peak corresponding to fundamental sloshing frequency is considerably reduced. Frequency tuning of one tube with sloshing reduces the contribution of sloshing to response of other tubes considerably though the response of the tuned tube increases many times. However, it may not always be possible to bring down the sloshing response of the tubes to desired level due to practical limitations of introducing sufficient damping in the tuned tube.

CONCLUSIONS

An efficient procedure is presented for dynamic response analysis of horizontal tube array in partially filled calandria including hydrodynamic interaction effects. The procedure is general enough to consider transfer of energy between the fluid-coupled tubes, and effects of moderator sloshing on the magnitude and the distribution of hydrodynamic forces. Use of semi-analytical approach for the evaluation of added hydrodynamic mass, damping and excitation terms due to interacting fluid leads to considerable economy in the computations.

The capabilities of the procedure developed have been demonstrated by presenting numerical results for a typical calandria with a large tube array to harmonic ground excitation in both the horizontal and the vertical directions. It has been demonstrated that because of phase lag between motions of tubes and calandria, and due to unequal cross-responses of tubes, the conventional added mass approach fails to represent behavior of the tube array correctly. It is therefore necessary to consider the flexibility of all the tubes simultaneously along two directions to get the correct response. The procedure developed can accommodate the added damping effects due to hydrodynamic interaction. However, the equivalent surface damping coefficient needs be established experimentally.

The possible use of a tuned damper tube is suggested for controlling sloshing effects for tube array in a calandria where tube frequencies and transverse sloshing frequencies are closely spaced. The tuned damper tube in the top layer of the array needs be designed to have natural frequency in air equal to that of first sloshing frequency of moderator in calandria. It has been demonstrated that the introduction of tuned damper tube reduces the contribution of sloshing to response of other tubes. The presence of surface damping in the tuned tube further brings down the response. If the tuned damper tube does not interfere with the functional requirements of the system, the proposed procedure can be effectively used to control unwarranted sloshing effects. However, it may not always be possible to achieve the desired damping in tuned damper tube due to functional constraints.

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APPENDIX: NOTATIONS

a_n^j	spatial distribution of surface acceleration for j-th tube
a_x, a_y,a_z	vault accelerations along x-, y- and z-axis, respectively
$\bar{A}_m^{xj}$, $\bar{A}_m^{zj}$	generalized co-ordinates associated with m-th Ritz function
C	acoustic velocity in moderator
E_{nm}^{xj}, E_{nm}^{zj}	coefficients in expansion series of the nth mode of the jth tube
H	length of the tubes and the calandria shell
$\mathbf{K}_j$	stiffness matrix of jth tube in generalized co-ordinates
$\bar{l}_n^j$	hydrodynamic forces in generalized modal co-ordinates
L_n^{xj}, L_n^{yj}, L_n^{zj}	generalized excitation terms in modal co-ordinates
$\bar{L}_n^{xja}$, $\bar{L}_n^{yja}$, $\bar{L}_n^{zja}$	added hydrodynamic excitation in the nth mode of jth tube
m^j	equivalent mass per unit length of the tube
M_n^j	generalized mass in modal co-ordinates
$\bar{M}_{nkl}^{ja}$	complex hydrodynamic mass on the jth tube in its nth mode due to motion of the kth tube in its lth mode
$\mathbf{M}_j$	mass matrix of jth tube in generalized co-ordinates
n_x^c, n_z^c	direction cosines of normal on the inside surface of calandria
n_x^j, n_z^j	direction cosines of normal on outside surface of jth tube
n_y^v	direction cosine of normal on inside surface of vault walls
N_j	number of modes considered for the jth tube
N_k	number of modes considered for the kth tube
N_m	number of terms considered in the semi-analytical process
N_t	number of horizontal tubes
$\bar{p}$	hydrodynamic pressure
$\bar{p}^j$	hydrodynamic pressure on outside surface of j-th tube
$\bar{p}_{0x}$, $\bar{p}_{0y}$, $\bar{p}_{0z}$	hydrodynamic pressures for rigid calandria and rigid tubes
$\bar{p}_n^j$	hydrodynamic pressure for excitation of jth tube in nth mode
q_j	equivalent surface damping coefficient for jth tube
r_x, r_y, r_z	components of vault acceleration along x-, y- and z-axis
R_j	response quantity for jth tube
S_{RR}	power spectral density function for response quantity R
S_{xx}^v, S_{xy}^v, … ,S_{zy}^v, S_{zz}^v	power spectral density functions for vault excitations
$\bar{u}^j$, $\bar{w}^j$	frequency response functions for jth tube axis displacements
$\mathbf{x}$	co-ordinate vector
$\bar{Y}_n^j$	generalized co-ordinate associated with nth mode of jth tube
α_m	$= 2m\pi/H$
$\bar{\phi}_n^{xj}$, $\bar{\phi}_n^{zj}$	displacement functions for nth mode of jth tube
Γ_c	calandria–moderator interface
Γ_f	free surface of the moderator
Γ_j	jth horizontal tube–moderator interface
Γ_v	vault walls–moderator interface
η_j	constant hysteretic damping factor for jth tube in air
η_s	damping factor for free surface
ρ	mass density of moderator
ω_n^j	natural frequency of jth tube in its nth mode in air
ω	excitation frequency

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